Ex. 5,4 p.264 # 1 à 10, algébriquement seulement, 11 à 20, 26 à 33 simplifie

Vérifie graphiquement la possibilité d'une identité. Ensuite, prouve chaque identité algébriquement.

1. $\sin\theta \sec\theta = \tan\theta$

$$\sin\theta \times \frac{1}{\cos\theta} = \tan\theta$$
$$\tan\theta = \tan\theta$$

2. $\cos\theta \operatorname{cosec}\theta = \operatorname{cotan}\theta$

$$\cos\theta \times \frac{1}{\sin\theta} = \operatorname{cotan}\theta$$
$$\operatorname{cotan}\theta = \operatorname{cotan}\theta$$

3. $\operatorname{cotan}\theta \sin\theta = \cos\theta$

$$\frac{\cos\theta}{\sin\theta} \times \sin\theta = \cos\theta$$
$$\cos\theta = \cos\theta$$

4. $\cos\theta + \tan\theta \sin\theta = \sec\theta$

$$\cos\theta + \frac{\sin\theta}{\cos\theta} \times \sin\theta = \sec\theta$$
$$\frac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \sec\theta$$
$$\frac{1}{\cos\theta} = \sec\theta$$
$$\sec\theta = \sec\theta$$

5. $\tan\theta + \operatorname{cotan}\theta = \sec\theta \operatorname{cosec}\theta$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \sec\theta \operatorname{cosec}\theta$$
$$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta} = \sec\theta \operatorname{cosec}\theta$$
$$\frac{1}{\cos\theta \sin\theta} = \sec\theta \operatorname{cosec}\theta$$
$$\sec\theta \operatorname{cosec}\theta = \sec\theta \operatorname{cosec}\theta$$

6. $\frac{\sin^2\theta}{\cos^2\theta} = \sec^2\theta - 1$

$$\frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$$
$$\frac{\sin^2\theta}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta}$$

7. $\operatorname{cotan}\theta + \tan\theta = \sec\theta \operatorname{cosec}\theta$

$$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \sec\theta \operatorname{cosec}\theta$$
$$\frac{\cos^2\theta + \sin^2\theta}{\cos\theta \sin\theta} = \sec\theta \operatorname{cosec}\theta$$
$$\frac{1}{\cos\theta \sin\theta} = \sec\theta \operatorname{cosec}\theta$$
$$\sec\theta \operatorname{cosec}\theta = \sec\theta \operatorname{cosec}\theta$$

8. $\frac{\cos\theta}{\sin\theta \operatorname{cotan}\theta} = 1$

$$\frac{\cos\theta}{\sin\theta \times \frac{\cos\theta}{\sin\theta}} = 1$$
$$\frac{\cos\theta}{\cos\theta} = 1$$
$$1 = 1$$

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9. $\dfrac{\sin\theta + \tan\theta}{\cos\theta + 1} = \tan\theta$

$$\frac{\sin\theta + \dfrac{\sin\theta}{\cos\theta}}{\cos\theta + 1} = \tan\theta$$
$$\frac{\sin\theta\cos\theta + \sin\theta}{\cos\theta} \times \frac{1}{\cos\theta + 1} = \tan\theta$$
$$\frac{\sin\theta(\cos\theta + 1)}{\cos\theta(\cos\theta + 1)} = \tan\theta$$
$$\tan\theta = \tan\theta$$

10. $\dfrac{\sec\theta}{\cot an\theta + \tan\theta} = \sin\theta$

$$\frac{1/\cos\theta}{\cos\theta/\sin\theta + \sin\theta/\cos\theta} = \sin\theta$$
$$\frac{1/\cos\theta}{\dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}} = \sin\theta$$
$$\frac{1}{\cos\theta} \times \frac{\sin\theta\cos\theta}{\cos^2\theta + \sin^2\theta} = \sin\theta$$
$$\sin\theta = \sin\theta$$

Pour chaque identité :

a) Montre qu'elle est vraie lorsque $\theta = 30°$ à l'aide de valeurs exactes.
b) Prouve algébriquement que l'équation est une identité.
c) Indique les restrictions, s'il y en a.

11. $\sin^4\theta - \cos^4\theta = 2\sin^2\theta - 1$

$$\sin^4 30° - \cos^4 30° = 2\sin^2 30° - 1$$
$$\left(\frac{1}{2}\right)^4 - \left(\frac{\sqrt{3}}{2}\right)^4 = 2\left(\frac{1}{2}\right)^2 - 1$$
$$\frac{1}{16} - \frac{9}{16} = \frac{2}{4} - 1$$
$$\frac{-8}{16} = \frac{-2}{4}$$
$$\frac{-1}{2} = \frac{-1}{2}$$

$$(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta) = 2\sin^2\theta - 1$$
$$(\sin^2\theta - \cos^2\theta)(1) = 2\sin^2\theta - 1$$
$$(\sin^2\theta - (1 - \sin^2\theta)) = 2\sin^2\theta - 1$$
$$(\sin^2\theta - 1 + \sin^2\theta) = 2\sin^2\theta - 1$$
$$2\sin^2\theta - 1 = 2\sin^2\theta - 1$$

12. $\sin\theta + \cos\theta\cot an\theta = \cos ec\theta$

$$\sin 30° + \cos 30° \cot an 30° = \cos ec 30°$$
$$\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}/2}{1/2}\right) = \frac{1}{1/2}$$
$$\frac{1}{2} + \frac{3}{2} = 2$$
$$\frac{4}{2} = 2$$
$$2 = 2$$

$$\sin\theta + \cos\theta\frac{\cos\theta}{\sin\theta} = \cos ec\theta$$
$$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta} = \cos ec\theta$$
$$\frac{1}{\sin\theta} = \cos ec\theta$$
$$\cos ec\theta = \cos ec\theta$$

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13. $\cos\theta(\operatorname{cosec}\theta - \sec\theta) = \operatorname{cotan}\theta - 1$

$$\cos 30^\circ(\operatorname{cosec}30^\circ - \sec 30^\circ) = \operatorname{cotan}30^\circ - 1$$
$$\left(\frac{\sqrt{3}}{2}\right)\left(\left(\frac{1}{1/2}\right) - \left(\frac{1}{\sqrt{3}/2}\right)\right) = \frac{\sqrt{3}/2}{1/2} - 1$$
$$\sqrt{3} - 1 = \sqrt{3} - 1$$

$$\cos\theta\left(\frac{1}{\sin\theta} - \frac{1}{\cos\theta}\right) = \operatorname{cotan}\theta - 1$$
$$\frac{\cos\theta}{\sin\theta} - \frac{\cos\theta}{\cos\theta} = \operatorname{cotan}\theta - 1$$
$$\operatorname{cotan}\theta - 1 = \operatorname{cotan}\theta - 1$$

14. $(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2 = 2$

$$(\sin 30^\circ - \cos 30^\circ)^2 + (\sin 30^\circ + \cos 30^\circ)^2 = 2$$
$$\left(\frac{1}{2} - \frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right)^2 = 2$$
$$\frac{1}{4} - \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} + \frac{3}{4} + \frac{1}{4} + \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} + \frac{3}{4} = 2$$
$$\frac{8}{4} = 2$$
$$2 = 2$$

$$(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2 = 2$$
$$\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = 2$$
$$2\sin^2\theta + 2\cos^2\theta = 2$$
$$2(\sin^2\theta + \cos^2\theta) = 2$$
$$2 = 2$$

15. $1 - \sin\theta\cos\theta\tan\theta = \cos^2\theta$

$$1 - \sin 30^\circ \cos 30^\circ \tan 30^\circ = \cos^2 30^\circ$$
$$1 - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1/2}{\sqrt{3}/2}\right) = \left(\frac{\sqrt{3}}{2}\right)^2$$
$$1 - \left(\frac{\sqrt{3}}{4}\right)\left(\frac{1}{2} \times \frac{2}{\sqrt{3}}\right) = \frac{3}{4}$$
$$1 - \frac{1}{4} = \frac{3}{4}$$
$$\frac{3}{4} = \frac{3}{4}$$

$$1 - \sin\theta\cos\theta\tan\theta = \cos^2\theta$$
$$1 - \sin\theta\cos\theta\frac{\sin\theta}{\cos\theta} = \cos^2\theta$$
$$1 - \sin^2\theta = \cos^2\theta$$
$$\cos^2\theta = \cos^2\theta$$

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16. $\dfrac{1+\cos\theta}{\sin\theta}=\dfrac{\sin\theta}{1-\cos\theta}$

$$\frac{1+\cos 30^\circ}{\sin 30^\circ}=\frac{\sin 30^\circ}{1-\cos 30^\circ}$$

$$\frac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\frac{\frac{1}{2}}{1-\frac{\sqrt{3}}{2}}$$

$$\frac{\frac{2+\sqrt{3}}{2}}{\frac{1}{2}}=\frac{\frac{1}{2}}{\frac{2-\sqrt{3}}{2}}$$

$$\frac{2+\sqrt{3}}{2}\times\frac{2}{1}=\frac{1}{2}\times\frac{2}{2-\sqrt{3}}$$

$$2+\sqrt{3}=\frac{1}{2-\sqrt{3}}\times\frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$2+\sqrt{3}=\frac{2+\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-3}$$

$$2+\sqrt{3}=2+\sqrt{3}$$

$$\frac{1+\cos\theta}{\sin\theta}=\frac{\sin\theta}{1-\cos\theta}$$

$$\frac{1+\cos\theta}{\sin\theta}\times\frac{1-\cos\theta}{1-\cos\theta}=\frac{\sin\theta}{1-\cos\theta}$$

$$\frac{1-\cos\theta+\cos\theta-\cos^2\theta}{\sin\theta(1-\cos\theta)}=\frac{\sin\theta}{1-\cos\theta}$$

$$\frac{1-\cos^2\theta}{\sin\theta(1-\cos\theta)}=\frac{\sin\theta}{1-\cos\theta}$$

$$\frac{\sin^2\theta}{\sin\theta(1-\cos\theta)}=\frac{\sin\theta}{1-\cos\theta}$$

$$\frac{\sin\theta}{1-\cos\theta}=\frac{\sin\theta}{1-\cos\theta}$$

17. $\dfrac{1+\tan\theta}{1+\cot an\theta}=\tan\theta$

$$\frac{1+\tan 30^\circ}{1+\cot an 30^\circ}=\tan 30^\circ$$

$$\frac{1+\frac{1/2}{\sqrt{3}/2}}{1+\frac{\sqrt{3}/2}{1/2}}=\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$$

$$\frac{1+\frac{1}{2}\times\frac{2}{\sqrt{3}}}{1+\frac{\sqrt{3}}{2}\times\frac{2}{1}}=\frac{1}{2}\times\frac{2}{\sqrt{3}}$$

$$\frac{1+\frac{1}{\sqrt{3}}}{1+\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$$\frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{1+\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$$\frac{\sqrt{3}+1}{\sqrt{3}}\times\frac{1}{1+\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$$\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$$\frac{1+\sin\theta/\cos\theta}{1+\cos\theta/\sin\theta}=\tan\theta$$

$$\frac{\frac{\cos\theta+\sin\theta}{\cos\theta}}{\frac{\sin\theta+\cos\theta}{\sin\theta}}=\tan\theta$$

$$\frac{\cos\theta+\sin\theta}{\cos\theta}\times\frac{\sin\theta}{\sin\theta+\cos\theta}=\tan\theta$$

$$\frac{\sin\theta}{\cos\theta}=\tan\theta$$

$$\tan\theta=\tan\theta$$

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18. $\dfrac{\cos\theta}{\sec\theta-1}+\dfrac{\cos\theta}{\sec\theta+1}=2\,\text{cotan}^2\theta$

$$\frac{\cos 30^\circ}{\sec 30^\circ-1}+\frac{\cos 30^\circ}{\sec 30^\circ+1}=2\,\text{cotan}^2 30^\circ$$

$$\frac{\sqrt{3}/2}{2/\sqrt{3}-1}+\frac{\sqrt{3}/2}{2/\sqrt{3}+1}=2\frac{\left(\frac{\sqrt{3}}{2}\right)^2}{\left(\frac{1}{2}\right)^2}$$

$$\frac{\sqrt{3}/2}{\frac{2-\sqrt{3}}{\sqrt{3}}}+\frac{\sqrt{3}/2}{\frac{2+\sqrt{3}}{\sqrt{3}}}=2\times\frac{3}{4}\times\frac{4}{1}$$

$$\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2-\sqrt{3}}+\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2+\sqrt{3}}=6$$

$$\frac{3}{4-2\sqrt{3}}+\frac{3}{4+2\sqrt{3}}=6$$

$$\frac{3\left(4+2\sqrt{3}\right)+3\left(4-2\sqrt{3}\right)}{\left(4-2\sqrt{3}\right)\left(4+2\sqrt{3}\right)}=6$$

$$\frac{12+6\sqrt{3}+12-6\sqrt{3}}{16-8\sqrt{3}+8\sqrt{3}-12}=6$$

$$\frac{24}{4}=6$$

$$6=6$$

$$\frac{\cos\theta}{\sec\theta-1}+\frac{\cos\theta}{\sec\theta+1}=2\,\text{cotan}^2\theta$$

$$\frac{\cos\theta(\sec\theta+1)+\cos\theta(\sec\theta-1)}{(\sec\theta-1)(\sec\theta+1)}=2\,\text{cotan}^2\theta$$

$$\frac{\cos\theta\sec\theta+\cos\theta+\cos\theta\sec\theta-\cos\theta}{\sec^2\theta+\sec\theta-\sec\theta-1}=2\,\text{cotan}^2\theta$$

$$\frac{2\cos\theta\sec\theta}{\sec^2\theta-1}=2\,\text{cotan}^2\theta$$

$$\frac{2\cos\theta\,\frac{1}{\cos\theta}}{\tan^2\theta}=2\,\text{cotan}^2\theta$$

$$\frac{2}{\tan^2\theta}=2\,\text{cotan}^2\theta$$

$$2\,\text{cotan}^2\theta=2\,\text{cotan}^2\theta$$

19. $\dfrac{\sec\theta}{\sin\theta}-\dfrac{\sin\theta}{\cos\theta}=\text{cotan}\,\theta$

$$\frac{\sec 30^\circ}{\sin 30^\circ}-\frac{\sin 30^\circ}{\cos 30^\circ}=\text{cotan}\,30^\circ$$

$$\frac{2/\sqrt{3}}{1/2}-\frac{1/2}{\sqrt{3}/2}=\sqrt{3}$$

$$\frac{2}{\sqrt{3}}\times\frac{2}{1}-\frac{1}{2}\times\frac{2}{\sqrt{3}}=\sqrt{3}$$

$$\frac{4}{\sqrt{3}}-\frac{1}{\sqrt{3}}=\sqrt{3}$$

$$\frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\sqrt{3}$$

$$\frac{3\sqrt{3}}{3}=\sqrt{3}$$

$$\sqrt{3}=\sqrt{3}$$

$$\frac{\sec\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}=\text{cotan}\,\theta$$

$$\frac{1/\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}=\text{cotan}\,\theta$$

$$\frac{1}{\cos\theta}\times\frac{1}{\sin\theta}-\frac{\sin\theta}{\cos\theta}=\text{cotan}\,\theta$$

$$\frac{1-\sin^2\theta}{\cos\theta\sin\theta}=\text{cotan}\,\theta$$

$$\frac{\cos^2\theta}{\cos\theta\sin\theta}=\text{cotan}\,\theta$$

$$\frac{\cos\theta}{\sin\theta}=\text{cotan}\,\theta$$

$$\text{cotan}\,\theta=\text{cotan}\,\theta$$

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20. $\dfrac{\tan\theta}{1+\tan\theta}=\dfrac{\sin\theta}{\sin\theta+\cos\theta}$

$$\frac{\tan 30^\circ}{1+\tan 30^\circ}=\frac{\sin 30^\circ}{\sin 30^\circ+\cos 30^\circ}$$

$$\frac{1/\sqrt{3}}{1+1/\sqrt{3}}=\frac{1/2}{1/2+\sqrt{3}/2}$$

$$\frac{1/\sqrt{3}}{\frac{\sqrt{3}+1}{\sqrt{3}}}=\frac{1/2}{\frac{1+\sqrt{3}}{2}}$$

$$\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{1+\sqrt{3}}=\frac{1}{2}\times\frac{2}{1+\sqrt{3}}$$

$$\frac{1}{1+\sqrt{3}}=\frac{1}{1+\sqrt{3}}$$

$$\frac{\sin\theta/\cos\theta}{1+\sin\theta/\cos\theta}=\frac{\sin\theta}{\sin\theta+\cos\theta}$$

$$\frac{\sin\theta/\cos\theta}{\frac{\cos\theta+\sin\theta}{\cos\theta}}=\frac{\sin\theta}{\sin\theta+\cos\theta}$$

$$\frac{\sin\theta}{\cos\theta}\times\frac{\cos\theta}{\cos\theta+\sin\theta}=\frac{\sin\theta}{\sin\theta+\cos\theta}$$

$$\frac{\sin\theta}{\cos\theta+\sin\theta}=\frac{\sin\theta}{\sin\theta+\cos\theta}$$

26. $\tan x\cos^2 x$

$$=\frac{\sin x}{\cos x}\times\cos^2 x$$

$$=\sin x\cos x$$

27. $\cos ec^2 x-\cot an^2 x$

$$=\frac{1}{\sin^2 x}-\frac{\cos^2 x}{\sin^2 x}$$

$$=\frac{1-\cos^2 x}{\sin^2 x}=\frac{\sin^2 x}{\sin^2 x}=1$$

28. $\dfrac{\sin x}{1+\cos x}+\dfrac{\cos x}{\sin x}$

$$=\frac{\sin^2 x+\cos x(1+\cos x)}{(1+\cos x)\sin x}$$

$$=\frac{\sin^2 x+\cos x+\cos^2 x}{(1+\cos x)\sin x}$$

$$=\frac{1+\cos x}{(1+\cos x)\sin x}=\frac{1}{\sin x}$$

29. $(1+\sin x)^2+\cos^2 x$

$$=1+2\sin x+\sin^2 x+\cos^2 x$$

$$=1+2\sin x+1$$

$$=2+2\sin x$$

$$=2(1+\sin x)$$

30. $2(\cos ec^2 x-\cot an^2 x)$

$$=2\left(\frac{1}{\sin^2 x}-\frac{\cos^2 x}{\sin^2 x}\right)$$

$$=2\left(\frac{1-\cos^2 x}{\sin^2 x}\right)$$

$$=2\left(\frac{\sin^2 x}{\sin^2 x}\right)=2$$

31. $\dfrac{\cos ec x\times\sec x}{\cot an x}$

$$=\frac{\frac{1}{\sin x}\times\frac{1}{\cos x}}{\frac{\cos x}{\sin x}}$$

$$=\frac{1}{\sin x\cos x}\times\frac{\sin x}{\cos x}$$

$$=\frac{1}{\cos^2 x}=\sec^2 x$$

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32. $\frac{\sin^2 x}{\cos^2 x} + \sin x \cos ec x$

$= \frac{\sin^2 x}{\cos^2 x} + \sin x \frac{1}{\sin x}$

$= \frac{\sin^2 x}{\cos^2 x} + 1$

$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x} = \sec^2 x$

33. $\frac{\tan^2 x - \sin^2 x}{\tan^2 x \sin^2 x}$

$= \frac{\tan^2 x}{\tan^2 x \sin^2 x} - \frac{\sin^2 x}{\tan^2 x \sin^2 x}$

$= \frac{1}{\sin^2 x} - \frac{1}{\tan^2 x}$

$= \frac{1}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}$

$= \frac{1 - \cos^2 x}{\sin^2 x} = \frac{\sin^2 x}{\sin^2 x} = 1$