Math 30411

Module 6 – Trigonométrie – partie 2



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5.2 Isole x dans chaque équation, $0° \le x < 360°$

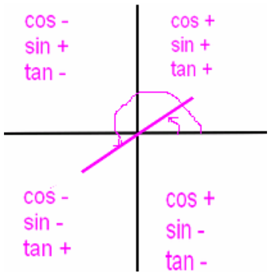
9. $\sqrt{3}\tan 2x = 1$

$\tan 2x = \frac{1}{\sqrt{3}}$

$2x = 30° + 180°n$

$x = 15° + 90°n$

$x = 15°, 105°, 195°, 285°$




11. $\sin 3x + 1 = 1$

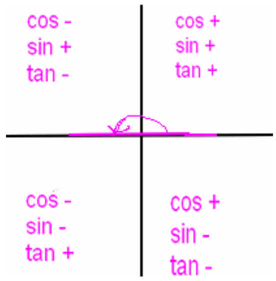
$\sin 3x = 0$

$3x = 0° + 180°n$

$x = 0° + 60°n$

$x = 0°, 60°, 120°,$

$180°, 240°, 300°$

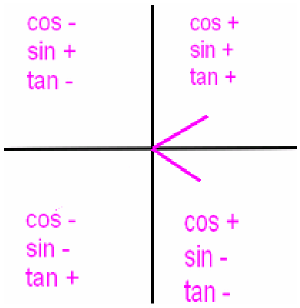



13. $2\cos x - \sqrt{3} = 0$

$\cos x = \frac{\sqrt{3}}{2}$

$x = 30° + 360°n, 330° + 360°n$

$x = 30°, 330°$



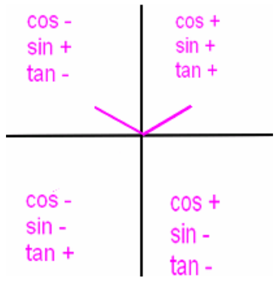

15. $2\sin\frac{1}{2}x = \sqrt{3}$

$\sin\frac{1}{2}x = \frac{\sqrt{3}}{2}$

$\frac{1}{2}x = 60° + 360°n, 120° + 360°n$

$x = 120° + 720°n, 240° + 720°n$

$x = 120°, 240°$




Isole $\theta$ dans chaque équation, pour l'intervalle $0 \le x < 2\pi$, puis indique une solution générale.

17. $\sin^2\theta - \frac{3}{4} = 0$

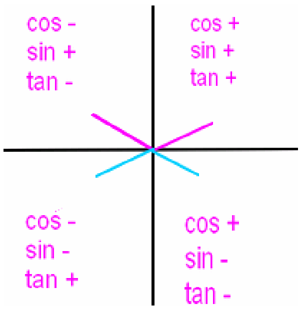
$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \frac{\pm\sqrt{3}}{2}$

$\sin\theta = \frac{\sqrt{3}}{2}, \sin\theta = \frac{-\sqrt{3}}{2}$

$\theta = \frac{\pi}{3} + \pi, \frac{2\pi}{3} + \pi$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$




19. $\tan^2\theta - 1 = 0$

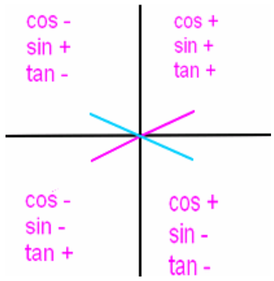
$\tan^2\theta = 1$

$\tan\theta = \pm 1$

$\tan\theta = 1, \tan\theta = -1$

$\theta = \frac{\pi}{4} + n\pi, \frac{3\pi}{4} + n\pi$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$




21. $4\cos^2\theta + 2\cos\theta - 2 = 0$

$4\cos^2\theta + 4\cos\theta - 2\cos\theta - 2 = 0$

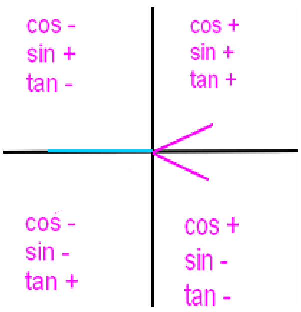
$4\cos\theta(\cos\theta + 1) - 2(\cos\theta + 1) = 0$

$(\cos\theta + 1)(4\cos\theta - 2) = 0$

$\cos\theta = -1, \cos\theta = \frac{1}{2}$

$\theta = \pi + 2\pi n, \frac{\pi}{3} + 2\pi n, \frac{5\pi}{3} + 2\pi n$

$\theta = \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

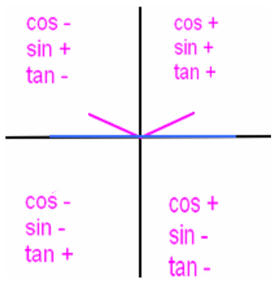



23. $\sin\theta\left(\sin\theta - \frac{\sqrt{3}}{2}\right) = 0$

$\sin\theta = 0, \sin\theta = \frac{\sqrt{3}}{2}$

$\theta = 0 + \pi n, \frac{\pi}{3} + 2\pi, \frac{2\pi}{3} + 2\pi$

$\theta = 0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}$



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5.4 Vérifie graphiquement si l'équation semble être une identité. Ensuite, prouve chaque identité algébriquement.

27. $\csc^2 x(1-\cos^2 x)=1$

$$\frac{1}{\sin^2 x}\times\sin^2 x=1$$
$$1=1$$

29. $\dfrac{\sin^2 x+\cos^2 x}{\sec x}=\cos x$

$$\frac{1}{1/\cos x}=\cos x$$
$$1\times\cos x=\cos x$$
$$\cos x=\cos x$$

31. $\dfrac{\cos x+1}{\sin x+\tan x}=\cot x$

$$\frac{\cos x+1}{\sin x+\frac{\sin x}{\cos x}}=\cot x$$
$$\frac{\cos x+1}{\frac{\sin x\cos x+\sin x}{\cos x}}=\cot x$$
$$\frac{\cos x+1}{\frac{\sin x(\cos x+1)}{\cos x}}=\cot x$$
$$(\cos x+1)\times\frac{\cos x}{\sin x(\cos x+1)}=\cot x$$
$$\frac{\cos x}{\sin x}=\cot x$$
$$\cot x=\cot x$$

Pour chaque identité

a) montre qu'elle est vraie lorsque $\theta=30^\circ$ à l'aide de valeurs exactes.
b) Prouve le résultat algébriquement
c) Indique les restrictions, s'il y en a.

33. $\cos\theta+\cos\theta\tan^2\theta=\sec\theta$

$$\cos 30^\circ+\cos 30^\circ\tan^2 30^\circ=\sec 30^\circ$$
$$\frac{\sqrt3}{2}+\frac{\sqrt3}{2}\times\left(\frac{1}{\sqrt3}\right)^2=\frac{2}{\sqrt3}$$
$$\frac{\sqrt3}{2}+\frac{\sqrt3}{6}=\frac{2}{\sqrt3}$$
$$\frac{3\sqrt3+\sqrt3}{6}=\frac{2}{\sqrt3}$$
$$\frac{4\sqrt3}{6}=\frac{2}{\sqrt3}$$
$$\frac{2\sqrt3}{3}\times\frac{\sqrt3}{\sqrt3}=\frac{2}{\sqrt3}$$
$$\frac{2\times3}{3\sqrt3}=\frac{2}{\sqrt3}$$
$$\frac{2}{\sqrt3}=\frac{2}{\sqrt3}$$

$$\cos\theta+\cos\theta\frac{\sin^2\theta}{\cos^2\theta}=\sec\theta$$
$$\cos\theta+\frac{\sin^2\theta}{\cos\theta}=\sec\theta$$
$$\frac{\cos^2\theta+\sin^2\theta}{\cos\theta}=\sec\theta$$
$$\frac{1}{\cos\theta}=\sec\theta$$
$$\sec\theta=\sec\theta$$

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35. $\dfrac{\sin\theta+\tan\theta}{\cos\theta+1}=\tan\theta$

$$\frac{\sin 30^\circ+\tan 30^\circ}{\cos 30^\circ+1}=\tan 30^\circ$$
$$\frac{\frac{1}{2}+\frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{2}+1}=\frac{1}{\sqrt{3}}$$
$$\frac{\frac{\sqrt{3}+2}{2\sqrt{3}}}{\frac{\sqrt{3}+2}{2}}=\frac{1}{\sqrt{3}}$$
$$\frac{\sqrt{3}+2}{2\sqrt{3}}\times\frac{2}{\sqrt{3}+2}=\frac{1}{\sqrt{3}}$$
$$\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$$\frac{\sin\theta+\frac{\sin\theta}{\cos\theta}}{\cos\theta+1}=\tan\theta$$
$$\frac{\frac{\cos\theta\sin\theta+\sin\theta}{\cos\theta}}{\cos\theta+1}=\tan\theta$$
$$\frac{\sin\theta(\cos\theta+1)}{\cos\theta}\times\frac{1}{\cos\theta+1}=\tan\theta$$
$$\tan\theta=\tan\theta$$

À partir de ta connaissance des identités trigonométriques et des procédés algébriques, simplifie chaque expression. Ensuite, utilise une calculatrice à affichage graphique pour vérifier si le résultat est équivalent à l'expression originale.

37. $\sec^2 x-\tan^2 x$

$$=\frac{1}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}$$
$$=\frac{1-\sin^2 x}{\cos^2 x}$$
$$=\frac{\cos^2 x}{\cos^2 x}$$
$$=1$$

39. $(1+\cos x)^2+\sin^2 x$

$$=1+\cos x+\cos x+\cos^2 x+\sin^2 x$$
$$=1+2\cos x+1$$
$$=2+2\cos x$$
$$=2(1+\cos x)$$

5.5. Indique la valeur exacte de chaque expression.

41. $\sin 100^\circ\cos 20^\circ+\cos 20^\circ\sin 100^\circ$

$$=\sin(100^\circ+20^\circ)$$
$$=\sin 120^\circ$$
$$=\frac{\sqrt{3}}{2}$$

43. $\cos\dfrac{\pi}{3}\cos\dfrac{\pi}{6}-\sin\dfrac{\pi}{3}\sin\dfrac{\pi}{6}$

$$=\cos\left(\frac{\pi}{3}+\frac{\pi}{6}\right)$$
$$=\cos\left(\frac{\pi}{2}\right)$$
$$=0$$

Exprime chaque énoncé sous la forme d'une fonction trigonométrique simple.

45. $\cos^2\dfrac{\pi}{4}-\sin^2\dfrac{\pi}{4}$

$$=\cos 2\left(\frac{\pi}{4}\right)$$
$$=\cos\frac{\pi}{2}$$

47. $2\sin 75^\circ\cos 75^\circ$

$$=\sin(75^\circ+75^\circ)$$
$$=\sin 150^\circ$$

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49. Prouve chaque équation algébriquement

a) $\cos(A+B)\cos(A-B)=\cos^2 A-\sin^2 B$

$$(\cos A\cos B-\sin A\sin B)(\cos A\cos B+\sin A\sin B)=\cos^2 A-\sin^2 B$$
$$\cos^2 A\cos^2 B-\cos A\cos B\sin A\sin B+\cos A\cos B\sin A\sin B-\sin^2 A\sin^2 B=\cos^2 A-\sin^2 B$$
$$\cos^2 A(1-\sin^2 B)-(1-\cos^2 A)\sin^2 B=\cos^2 A-\sin^2 B$$
$$\cos^2 A-\cos^2 A\sin^2 B-\sin^2 B+\cos^2 A\sin^2 B=\cos^2 A-\sin^2 B$$
$$\cos^2 A-\sin^2 B=\cos^2 A-\sin^2 B$$

b) $\dfrac{1+\cos 2\theta}{\sin 2\theta}=\cot an\theta$

$$\frac{1+\cos\theta\cos\theta-\sin\theta\sin\theta}{2\sin\theta\cos\theta}=\cotan\theta$$
$$\frac{1+\cos^2\theta-\sin^2\theta}{2\sin\theta\cos\theta}=\cotan\theta$$
$$\frac{1-\sin^2\theta+\cos^2\theta}{2\sin\theta\cos\theta}=\cotan\theta$$
$$\frac{\cos^2\theta+\cos^2\theta}{2\sin\theta\cos\theta}=\cotan\theta$$
$$\frac{2\cos^2\theta}{2\sin\theta\cos\theta}=\cotan\theta$$
$$\frac{\cos\theta}{\sin\theta}=\cotan\theta$$
$$\cotan\theta=\cotan\theta$$