

EXPRESSIONS RATIONNELLES 1

1. Simplifie les expressions rationnelles et indique les restrictions.

a)
$$\frac{7}{x^2 - 9} \div \frac{x + 4}{x + 3}$$

$$= \frac{7}{(x - 3)(x + 3)} \times \frac{x + 3}{x + 4}$$

$$= \frac{7}{(x - 3)(x + 4)}; x \neq 3, -3, -4$$

b)
$$\frac{x^2 + 1}{x^3 - 3x^2 + x - 3}$$

$$= \frac{x^2 + 1}{(x - 3)(x^2 + 1)}$$

$$= \frac{1}{(x - 3)}; x \neq 3$$

$x = 3$	1	-3	1	-3
+		3	0	3
	1	0	1	0

c)
$$\frac{-5}{16 - x^2} \div \frac{10}{x - 4}$$

$$= \frac{-5}{-(x^2 - 16)} \times \frac{x - 4}{10}$$

$$= \frac{-5}{-(x - 4)(x + 4)} \times \frac{x - 4}{10}$$

$$= \frac{1}{2(x + 4)}; x \neq 4, -4$$

d)
$$\frac{x + 4}{3 - x} \times \frac{x^2 - 9}{x^2 + 16}$$

$$= \frac{x + 4}{-(x - 3)} \times \frac{(x - 3)(x + 3)}{(x^2 + 16)}$$

$$= \frac{-(x + 4)(x + 3)}{(x^2 + 16)}; x \neq 3$$

e)
$$\frac{4x^2 - 9}{x^2 - 10x + 25} \div \frac{2x - 3}{x - 5}$$

$$= \frac{(2x - 3)(2x + 3)}{(x - 5)(x - 5)} \times \frac{x - 5}{2x - 3}$$

$$= \frac{(2x + 3)}{(x - 5)}; x \neq 5, \frac{3}{2}$$

f)
$$\frac{x^2 + x - 2}{x^2 + 4x + 4} \cdot \frac{x^2 + 5x + 6}{x^2 + 2x - 3} \div (x + 5)$$

$$= \frac{(x + 2)(x - 1)}{(x + 2)(x + 2)} \times \frac{(x + 2)(x + 3)}{(x + 3)(x - 1)} \times \frac{1}{x + 5}$$

$$= \frac{1}{x + 5}; x \neq -2, -3, 1, -5$$

g)
$$\frac{x^5 + 2x^2 - x + 2}{x^2 + 1}$$

$x^5 + 0x^3 + 2x^2 - x + 2$	$\frac{x^2 + 1}{x^3 - x + 2}$
$x^5 + x^3$	
$-x^3 + 2x^2 - x + 2$	
$-x^3 - x$	
$2x^2 + 2$	
$2x^2 + 2$	

$$= \frac{(x^2 + 1)(x^3 - x + 2)}{(x^2 + 1)}$$

$$= (x^3 - x + 2)$$

h)
$$\frac{x^3 - 2x^2 + 2x - 1}{x - 1}$$

$$= \frac{(x - 1)(x^2 - x + 1)}{(x - 1)}$$

$x = 1$	1	-2	2	-1
+		1	-1	1
	1	-1	1	0

$$= (x^2 - x + 1); x \neq 1$$

$$\begin{aligned}
 \text{i)} \quad & \frac{6x^2 + x - 1}{6x^2 + 5x + 1} \div \frac{3x^2 + 2x - 1}{3x^2 + 4x + 1} \\
 &= \frac{(6x+3)(6x-2)/6}{(6x+3)(6x+2)/6} \times \frac{(3x+3)(3x-1)/3}{(3x+3)(3x+1)/3} \\
 &= \frac{3(2x+1)2(3x-1)/6}{3(2x+1)2(3x+1)/6} \times \frac{3(x+1)(3x-1)/3}{3(x+1)(3x+1)/3} \\
 &= 1; x \neq \frac{-1}{2}, \frac{-1}{3}, -1, \frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{j)} \quad & \frac{x^3 - 3x^2 - 6x + 8}{8x - 32} \div \frac{x^2 + x - 2}{12x^2 - 16x + 4} \\
 & x = -2 \quad \begin{array}{r|rrrr} 1 & -3 & -6 & 8 \\ + & & -2 & 10 & -8 \\ \hline & 1 & -5 & 4 & 0 \end{array} \\
 &= \frac{(x+2)(x^2 - 5x + 4)}{8(x-4)} \times \frac{4(3x^2 - 4x + 1)}{(x+2)(x-1)} \\
 &= \frac{(x+2)(x-4)(x-1)}{8(x-4)} \times \frac{4(3x-3)(3x-1)/3}{(x+2)(x-1)} \\
 &= \frac{(x-1)(3x-1)}{2}; x \neq 4, -2, 1
 \end{aligned}$$

$$\begin{aligned}
 \text{k)} \quad & \frac{8x^3 - 1}{2x^3 - 5x^2 - 4x + 3} \times \frac{3x^2 - 6x - 9}{24x^2 + 12x + 6} \\
 & x = \frac{1}{2} \quad \begin{array}{r|rrrr} 8 & 0 & 0 & -1 \\ + & & 4 & 2 & 1 \\ \hline & 8 & 4 & 2 & 0 \\ \div 2 & 4 & 2 & 1 & 0 \end{array} \quad x = -1 \quad \begin{array}{r|rrrr} 2 & -5 & -4 & 3 \\ + & & -2 & 7 & -3 \\ \hline & 2 & -7 & 3 & 0 \end{array} \\
 &= \frac{(2x-1)(4x^2 + 2x + 1)}{(x+1)(2x^2 - 7x + 3)} \times \frac{3(x^2 - 2x - 3)}{6(4x^2 + 2x + 1)} \\
 &= \frac{(2x-1)}{(x+1)(2x-6)(2x-1)/2} \times \frac{3(x-3)(x+1)}{6} \\
 &= \frac{(2x-1)(4x^2 + 2x + 1)}{(x+1)2(x-3)(2x-1)/2} \times \frac{3(x-3)(x+1)}{6(4x^2 + 2x + 1)} \\
 &= \frac{1}{2}; x \neq -1, 3, \frac{1}{2}
 \end{aligned}$$

2. Écris les expressions suivantes sous forme d'une expression rationnelle et indique les restrictions.

$$\begin{aligned}
 \text{a)} \quad & \frac{x-3}{x-1} - \frac{4x-6}{x-1} \\
 &= \frac{(x-3) - (4x-6)}{x-1} \\
 &= \frac{x-3-4x+6}{x-1} \\
 &= \frac{-3x+3}{x-1} = \frac{-3(x-1)}{x-1} \\
 &= -3; x \neq 1
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & \frac{5}{x-3} + \frac{x+4}{3-x} \\
 &= \frac{5}{x-3} + \frac{-1(x+4)}{x-3} \\
 &= \frac{5-x-4}{x-3} \\
 &= \frac{-x+1}{x-3}; x \neq 3
 \end{aligned}$$

$$\begin{aligned}
 \text{c) } & \frac{x}{x^2 + 3x + 2} + \frac{1}{x^2 + x} \\
 &= \frac{x}{(x+2)(x+1)} + \frac{1}{x(x+1)} \\
 &= \frac{x(x) + 1(x+2)}{x(x+1)(x+2)} \\
 &= \frac{x^2 + x + 2}{x(x+1)(x+2)}; x \neq 0, -1, -2
 \end{aligned}$$

$$\begin{aligned}
 \text{d) } & \frac{x-3}{x^2 - x - 2} + \frac{4x+2}{x^2 - 4} \\
 &= \frac{x-3}{(x-2)(x+1)} + \frac{2(2x+1)}{(x-2)(x+2)} \\
 &= \frac{(x-3)(x+2) + 2(2x+1)(x+1)}{(x-2)(x+1)(x+2)} \\
 &= \frac{x^2 - 3x + 2x - 6 + 2(2x^2 + 2x + x + 1)}{(x-2)(x+1)(x+2)} \\
 &= \frac{x^2 - x - 6 + 4x^2 + 6x + 2}{(x-2)(x+1)(x+2)} \\
 &= \frac{5x^2 + 5x - 4}{(x-2)(x+1)(x+2)}; x \neq 2, -1, -2
 \end{aligned}$$

$$\begin{aligned}
 \text{e) } & 5 - \frac{4x^2 + 3}{x^2 - 2x - 8} \\
 &= \frac{5(x^2 - 2x - 8) - (4x^2 + 3)}{(x-4)(x+2)} \\
 &= \frac{5x^2 - 10x - 40 - 4x^2 - 3}{(x-4)(x+2)} \\
 &= \frac{x^2 - 10x - 43}{(x-4)(x+2)}; x \neq 4, -2
 \end{aligned}$$

$$\begin{aligned}
 \text{f) } & \frac{x^2 + 3x + 2}{x^3 - x^2 - 9x - 6} + \frac{x-2}{x^2 - 4} \\
 & \begin{array}{r|rrrr}
 x = -2 & 1 & -1 & -9 & -6 \\
 + & & -2 & 6 & 6 \\
 \hline
 & 1 & -3 & -3 & 0
 \end{array} \\
 &= \frac{(x+2)(x+1)}{(x+2)(x^2 - 3x - 3)} + \frac{(x-2)}{(x-2)(x+2)} \\
 &= \frac{(x+1)}{(x^2 - 3x - 3)} + \frac{1}{(x+2)} \\
 &= \frac{(x+1)(x+2) + (x^2 - 3x - 3)}{(x^2 - 3x - 3)(x+2)} \\
 &= \frac{x^2 + 2x + x + 2 + x^2 - 3x - 3}{(x^2 - 3x - 3)(x+2)} \\
 &= \frac{2x^2 - 1}{(x^2 - 3x - 3)(x+2)}; x \neq -2, 2
 \end{aligned}$$

$$\begin{aligned}
 \text{g) } & \frac{x}{x^2 + 3x - 18} + \frac{1}{x^2 + 6x} - \frac{1}{x^2 - 3x} \\
 &= \frac{x}{(x+6)(x-3)} + \frac{1}{x(x+6)} - \frac{1}{x(x-3)} \\
 &= \frac{x(x) + 1(x-3) - 1(x+6)}{x(x+6)(x-3)} \\
 &= \frac{x^2 + x - 3 - x - 6}{x(x+6)(x-3)} \\
 &= \frac{x^2 - 9}{x(x+6)(x-3)} \\
 &= \frac{(x-3)(x+3)}{x(x+6)(x-3)} \\
 &= \frac{x+3}{x(x+6)}; x \neq 0, -6, 3
 \end{aligned}$$

$$\begin{aligned}
 \text{h) } & \frac{2}{y+3} - \frac{y}{y-1} + \frac{y^2 + 2}{y^2 + 2y - 3} \\
 &= \frac{2}{y+3} - \frac{y}{y-1} + \frac{y^2 + 2}{(y+3)(y-1)} \\
 &= \frac{2(y-1) - y(y+3) + y^2 + 2}{(y+3)(y-1)} \\
 &= \frac{2y - 2 - y^2 - 3y + y^2 + 2}{(y+3)(y-1)} \\
 &= \frac{-y}{(y+3)(y-1)}; y \neq -3, 1
 \end{aligned}$$

$$\begin{aligned}
 \text{i) } & \frac{\frac{6x}{x^2 + x - 2}}{\frac{4}{x+2} - \frac{1}{x-1}} \\
 &= \frac{\frac{6x}{(x+2)(x-1)}}{\frac{4(x-1) - (x+2)}{(x+2)(x-1)}} \\
 &= \frac{\frac{6x}{(x+2)(x-1)}}{\frac{4x - 4 - x - 2}{(x+2)(x-1)}} = \frac{\frac{6x}{(x+2)(x-1)}}{\frac{3x - 6}{(x+2)(x-1)}} \\
 &= \frac{6x}{(x+2)(x-1)} \times \frac{(x+2)(x-1)}{3(x-2)} \\
 &= \frac{2x}{x-2}; x \neq -2, 1, 2
 \end{aligned}$$

$$j) \quad 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{x}}}}}$$

$$= 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{x+1}}}}$$

$$= 1 + \frac{1}{1 + \frac{1}{1 + \frac{x}{x+1}}}$$

$$= 1 + \frac{1}{1 + \frac{1}{x+1+x}}$$

$$= 1 + \frac{1}{1 + \frac{1}{2x+1}}$$

$$= 1 + \frac{1}{1 + \frac{1}{2x+1+x+1}}$$

$$= 1 + \frac{1}{1 + \frac{2x+1}{3x+2}}$$

$$= 1 + \frac{1}{3x+2+2x+1}$$

$$= 1 + \frac{3x+2}{5x+3} = \frac{5x+3+3x+2}{5x+3}$$

$$= \frac{8x+5}{5x+3}; x \neq 0, -1, -\frac{1}{2}, -\frac{2}{3}, -\frac{3}{5}$$

$$\begin{aligned} \text{k) } & \frac{\frac{x^2y}{x+y}}{xy} \\ &= \frac{x^2y}{x+y} \times \frac{1}{xy} \\ &= \frac{x}{x+y}; x \neq -y, 0; y \neq 0 \end{aligned}$$

$$\begin{aligned} \text{l) } & \frac{x^{-1} - 2}{x^{-1} + 3} \\ &= \frac{\frac{1}{x} - 2}{\frac{1}{x} + 3} \\ &= \frac{\frac{1-2x}{x}}{\frac{1+3x}{x}} \\ &= \frac{1-2x}{1+3x} \\ &= \frac{1-2x}{x} \times \frac{x}{1+3x} \\ &= \frac{1-2x}{1+3x}; x \neq 0, \frac{-1}{3} \end{aligned}$$