Omnimaths 12, pages 264-265, nos 1-15, 17-21, 23, 35b, 36(simplifie).

Prouve chaque identité algébriquement.

1. $\sin\theta \sec\theta = \tan\theta$

$$\sin\theta \times \frac{1}{\cos\theta} = \tan\theta$$
$$\tan\theta = \tan\theta$$

2. $\cos\theta \operatorname{cosec}\theta = \cot an\theta$

$$\cos\theta \times \frac{1}{\sin\theta} = \cot an\theta$$
$$\cot an\theta = \cot an\theta$$

3. $\cot an\theta \sin\theta = \cos\theta$

$$\frac{\cos\theta}{\sin\theta} \times \sin\theta = \cos\theta$$
$$\cos\theta = \cos\theta$$

4. $\cos\theta + \tan\theta \sin\theta = \sec\theta$

$$\cos\theta + \frac{\sin\theta}{\cos\theta} \times \sin\theta = \sec\theta$$
$$\frac{\cos^2\theta + \sin^2\theta}{\cos\theta} = \sec\theta$$
$$\frac{1}{\cos\theta} = \sec\theta$$
$$\sec\theta = \sec\theta$$

5. $\tan\theta + \cot an\theta = \sec\theta \operatorname{cosec}\theta$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \sec\theta \operatorname{cosec}\theta$$
$$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta} = \sec\theta \operatorname{cosec}\theta$$
$$\frac{1}{\cos\theta \sin\theta} = \sec\theta \operatorname{cosec}\theta$$
$$\sec\theta \operatorname{cosec}\theta = \sec\theta \operatorname{cosec}\theta$$

6. $\frac{\sin^2\theta}{\cos^2\theta} = \sec^2\theta - 1$

$$\frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$$
$$\frac{\sin^2\theta}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta}$$

7. $\cot an\theta + \tan\theta = \sec\theta \operatorname{cosec}\theta$

$$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \sec\theta \operatorname{cosec}\theta$$
$$\frac{\cos^2\theta + \sin^2\theta}{\cos\theta \sin\theta} = \sec\theta \operatorname{cosec}\theta$$
$$\frac{1}{\cos\theta \sin\theta} = \sec\theta \operatorname{cosec}\theta$$
$$\sec\theta \operatorname{cosec}\theta = \sec\theta \operatorname{cosec}\theta$$

8. $\frac{\cos\theta}{\sin\theta \cot an\theta} = 1$

$$\frac{\cos\theta}{\sin\theta \times \frac{\cos\theta}{\sin\theta}} = 1$$
$$\frac{\cos\theta}{\cos\theta} = 1$$
$$1 = 1$$

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9. $\dfrac{\sin\theta+\tan\theta}{\cos\theta+1}=\tan\theta$

$$\frac{\sin\theta+\dfrac{\sin\theta}{\cos\theta}}{\cos\theta+1}=\tan\theta$$

$$\frac{\sin\theta\cos\theta+\sin\theta}{\cos\theta}\times\frac{1}{\cos\theta+1}=\tan\theta$$

$$\frac{\sin\theta\left(\cos\theta+1\right)}{\cos\theta\left(\cos\theta+1\right)}=\tan\theta$$

$$\tan\theta=\tan\theta$$

10. $\dfrac{\sec\theta}{\cot an\theta+\tan\theta}=\sin\theta$

$$\frac{\dfrac{1}{\cos\theta}}{\dfrac{\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{\cos\theta}}=\sin\theta$$

$$\frac{\dfrac{1}{\cos\theta}}{\dfrac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}}=\sin\theta$$

$$\frac{1}{\cos\theta}\times\frac{\sin\theta\cos\theta}{\cos^2\theta+\sin^2\theta}=\sin\theta$$

$$\sin\theta=\sin\theta$$

11. $\sin^4\theta-\cos^4\theta=2\sin^2\theta-1$

$$\left(\sin^2\theta-\cos^2\theta\right)\left(\sin^2\theta+\cos^2\theta\right)=2\sin^2\theta-1$$

$$\left(\sin^2\theta-\cos^2\theta\right)(1)=2\sin^2\theta-1$$

$$\left(\sin^2\theta-\left(1-\sin^2\theta\right)\right)=2\sin^2\theta-1$$

$$\left(\sin^2\theta-1+\sin^2\theta\right)=2\sin^2\theta-1$$

$$2\sin^2\theta-1=2\sin^2\theta-1$$

12. $\sin\theta+\cos\theta\cot an\theta=\cos ec\theta$

$$\sin\theta+\cos\theta\frac{\cos\theta}{\sin\theta}=\cos ec\theta$$

$$\frac{\sin^2\theta+\cos^2\theta}{\sin\theta}=\cos ec\theta$$

$$\frac{1}{\sin\theta}=\cos ec\theta$$

$$\cos ec\theta=\cos ec\theta$$

13. $\cos\theta\left(\cos ec\theta-\sec\theta\right)=\cot an\theta-1$

$$\cos\theta\left(\frac{1}{\sin\theta}-\frac{1}{\cos\theta}\right)=\cot an\theta-1$$

$$\frac{\cos\theta}{\sin\theta}-\frac{\cos\theta}{\cos\theta}=\cot an\theta-1$$

$$\cot an\theta-1=\cot an\theta-1$$

14. $(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2 = 2$

$$(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2 = 2$$
$$\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = 2$$
$$2\sin^2\theta + 2\cos^2\theta = 2$$
$$2(\sin^2\theta + \cos^2\theta) = 2$$
$$2 = 2$$

15. $1 - \sin\theta\cos\theta\tan\theta = \cos^2\theta$

$$1 - \sin\theta\cos\theta\tan\theta = \cos^2\theta$$
$$1 - \sin\theta\cos\theta\frac{\sin\theta}{\cos\theta} = \cos^2\theta$$
$$1 - \sin^2\theta = \cos^2\theta$$
$$\cos^2\theta = \cos^2\theta$$

17. $\dfrac{1+\tan\theta}{1+\cot an\theta} = \tan\theta$

$$\frac{1 + \sin\theta/\cos\theta}{1 + \cos\theta/\sin\theta} = \tan\theta$$
$$\frac{\cos\theta + \sin\theta}{\dfrac{\cos\theta}{\sin\theta + \cos\theta}} = \tan\theta$$
$$\sin\theta$$
$$\frac{\cos\theta + \sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta + \cos\theta} = \tan\theta$$
$$\frac{\sin\theta}{\cos\theta} = \tan\theta$$
$$\tan\theta = \tan\theta$$

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18. $\dfrac{\cos\theta}{\sec\theta-1}+\dfrac{\cos\theta}{\sec\theta+1}=2\cot an^2\theta$

$$\frac{\cos\theta}{\sec\theta-1}+\frac{\cos\theta}{\sec\theta+1}=2\cot an^2\theta$$
$$\frac{\cos\theta(\sec\theta+1)+\cos\theta(\sec\theta-1)}{(\sec\theta-1)(\sec\theta+1)}=2\cot an^2\theta$$
$$\frac{\cos\theta\sec\theta+\cos\theta+\cos\theta\sec\theta-\cos\theta}{\sec^2\theta+\sec\theta-\sec\theta-1}=2\cot an^2\theta$$
$$\frac{2\cos\theta\sec\theta}{\sec^2\theta-1}=2\cot an^2\theta$$
$$\frac{2\cos\theta\dfrac{1}{\cos\theta}}{\tan^2\theta}=2\cot an^2\theta$$
$$\frac{2}{\tan^2\theta}=2\cot an^2\theta$$
$$2\cot an^2\theta=2\cot an^2\theta$$

19. $\dfrac{\sec\theta}{\sin\theta}-\dfrac{\sin\theta}{\cos\theta}=\cot an\theta$

$$\frac{\sec\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}=\cot an\theta$$
$$\frac{1}{\cos\theta}\Big/\frac{\sin\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}=\cot an\theta$$
$$\frac{1}{\cos\theta}\times\frac{1}{\sin\theta}-\frac{\sin\theta}{\cos\theta}=\cot an\theta$$
$$\frac{1-\sin^2\theta}{\cos\theta\sin\theta}=\cot an\theta$$
$$\frac{\cos^2\theta}{\cos\theta\sin\theta}=\cot an\theta$$
$$\frac{\cos\theta}{\sin\theta}=\cot an\theta$$
$$\cot an\theta=\cot an\theta$$

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20. $\dfrac{\tan\theta}{1+\tan\theta} = \dfrac{\sin\theta}{\sin\theta+\cos\theta}$

$$\frac{\sin\theta/\cos\theta}{1+\sin\theta/\cos\theta} = \frac{\sin\theta}{\sin\theta+\cos\theta}$$

$$\frac{\sin\theta/\cos\theta}{\dfrac{\cos\theta+\sin\theta}{\cos\theta}} = \frac{\sin\theta}{\sin\theta+\cos\theta}$$

$$\frac{\sin\theta}{\cos\theta}\times\frac{\cos\theta}{\cos\theta+\sin\theta} = \frac{\sin\theta}{\sin\theta+\cos\theta}$$

$$\frac{\sin\theta}{\cos\theta+\sin\theta} = \frac{\sin\theta}{\sin\theta+\cos\theta}$$

21. $\sin^2 x\sec^2 x = \sec^2 x - 1$

$$\sin^2 x\times\frac{1}{\cos^2 x} = \sec^2 x - 1$$

$$\tan^2 x = \sec^2 x - 1$$

$$\sec^2 x - 1 = \sec^2 x - 1$$

23. $\cot an^2 x\left(\sec^2 x - 1\right) = 1$

$$\frac{\cos^2 x}{\sin^2 x}\left(\frac{1}{\cos^2 x} - 1\right) = 1$$

$$\frac{\cos^2 x}{\sin^2 x\cos^2 x} - \frac{\cos^2 x}{\sin^2 x} = 1$$

$$\frac{1-\cos^2 x}{\sin^2 x} = 1$$

$$\frac{\sin^2 x}{\sin^2 x} = 1$$

$$1 = 1$$

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35. Technologie : soit l'équation $\frac{\cos ecB + co\tan B}{\tan B + \sin B} = cot\, anB\cos ecB$.

b) Prouve l'identité algébriquement.

$$\frac{\frac{1}{\sin B} + \frac{\cos B}{\sin B}}{\frac{\sin B}{\cos B} + \sin B} = cot\, anB\cos ecB$$

$$\frac{\frac{1+\cos B}{\sin B}}{\frac{\sin B + \cos B\sin B}{\cos B}} = cot\, anB\cos ecB$$

$$\frac{\frac{1+\cos B}{\sin B}}{\frac{\sin B(1+\cos B)}{\cos B}} = cot\, anB\cos ecB$$

$$\frac{1+\cos B}{\sin B} \times \frac{\cos B}{\sin B(1+\cos B)} = cot\, anB\cos ecB$$

$$cot\, anB\cos ecB = cot\, anB\cos ecB$$

36 Simplifie $f(x) = \frac{\cos x}{1+\sin x} + \frac{\cos x}{1-\sin x}$.

$$f(x) = \frac{\cos x(1-\sin x) + \cos x(1+\sin x)}{(1+\sin x)(1-\sin x)}$$

$$f(x) = \frac{\cos x(1-\sin x+1+\sin x)}{(1-\sin^2 x)}$$

$$f(x) = \frac{2\cos x}{(\cos^2 x)} = \frac{2}{\cos x} = 2\sec x$$