

TRIGONOMÉTRIE

1. Simplifie

a) $\sqrt{500}$

$$\begin{array}{r|l} 500 & 2 \\ 250 & 2 \\ 125 & 5 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array}$$

$$2 \times 5\sqrt{5}$$

$$= 10\sqrt{5}$$

b) $\sqrt{32}$

$$\begin{array}{r|l} 32 & 2 \\ 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array}$$

$$2 \times 2\sqrt{2}$$

$$= 4\sqrt{2}$$

c) $\sqrt{98}$

$$\begin{array}{r|l} 98 & 2 \\ 49 & 7 \\ 7 & 7 \\ 1 & \end{array}$$

$$= 7\sqrt{2}$$

d) $\sqrt{72}$

$$\begin{array}{r|l} 72 & 2 \\ 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

$$2 \times 3\sqrt{2}$$

$$= 6\sqrt{2}$$

e) $\sqrt{243}$

$$\begin{array}{r|l} 243 & 3 \\ 81 & 3 \\ 27 & 3 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

$$3 \times 3\sqrt{3}$$

$$= 9\sqrt{3}$$

f) $\sqrt{24}$

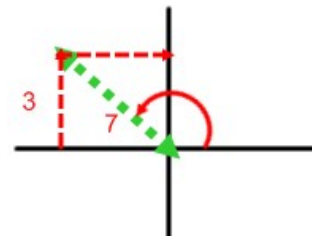
$$\begin{array}{r|l} 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array}$$

$$= 2\sqrt{2 \times 3}$$

$$= 2\sqrt{6}$$

2. Pour les angles θ mesurés en position standard, détermine la valeur exacte de tous les rapports trigonométriques.

a) Côté terminal dans le quadrant 2, $\sin \theta = \frac{3}{7}$



$$7^2 = x^2 + 3^2$$

$$49 - 9 = x^2$$

$$x = -\sqrt{40} = -2\sqrt{10}$$

$$\sin \theta = \frac{3}{7}$$

$$\cos \theta = \frac{-2\sqrt{10}}{7}$$

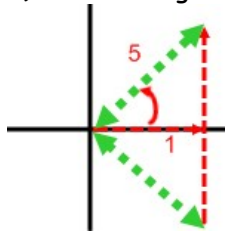
$$\tan \theta = \frac{-3}{2\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{-3\sqrt{10}}{20}$$

$$\operatorname{cosec} \theta = \frac{7}{3}$$

$$\sec \theta = \frac{-7}{2\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{-7\sqrt{10}}{20}$$

$$\operatorname{cotan} \theta = \frac{-2\sqrt{10}}{3}$$

b) θ est un angle aigu, $\sec \theta = 5$



$$5^2 = 1^2 + y^2$$

$$25 - 1 = y^2$$

$$x = \sqrt{24} = 2\sqrt{6}$$

$$\sin \theta = \frac{2\sqrt{6}}{5}$$

$$\cos \theta = \frac{1}{5}$$

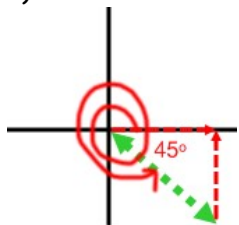
$$\tan \theta = \frac{2\sqrt{6}}{1}$$

$$\operatorname{cosec} \theta = \frac{5}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{5\sqrt{6}}{12}$$

$$\sec \theta = \frac{5}{1}$$

$$\operatorname{cotan} \theta = \frac{1}{2\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{12}$$

c) $\theta = 675^\circ$



$$\sin \theta = \frac{-\sqrt{2}}{2}$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

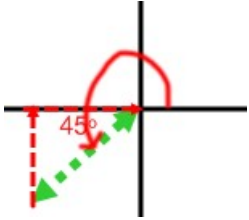
$$\tan \theta = -1$$

$$\operatorname{cosec} \theta = \frac{-2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$$

$$\sec \theta = \frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\operatorname{cotan} \theta = -1$$

d) $\theta \in [180^\circ, 360^\circ], \sin \theta = \cos \theta$



$$\sin \theta = \frac{-\sqrt{2}}{2}$$

$$\cos \theta = \frac{-\sqrt{2}}{2}$$

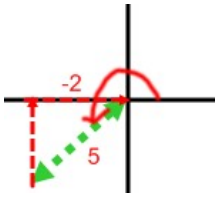
$$\tan \theta = 1$$

$$\operatorname{cosec} \theta = \frac{-2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$$

$$\sec \theta = \frac{-2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$$

$$\operatorname{cotan} \theta = 1$$

e) $\cos \theta = \frac{-2}{5}, \tan \theta > 0$



$$5^2 = (-2)^2 + y^2$$

$$25 - 4 = y^2$$

$$y = -\sqrt{21}$$

$$\sin \theta = \frac{-\sqrt{21}}{5}$$

$$\cos \theta = \frac{-2}{5}$$

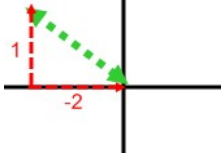
$$\tan \theta = \frac{\sqrt{21}}{2}$$

$$\operatorname{cosec} \theta = \frac{-5}{\sqrt{21}} \times \frac{\sqrt{21}}{\sqrt{21}} = \frac{-5\sqrt{21}}{21}$$

$$\sec \theta = \frac{-5}{2}$$

$$\operatorname{cotan} \theta = \frac{2}{\sqrt{21}} \times \frac{\sqrt{21}}{\sqrt{21}} = \frac{2\sqrt{21}}{21}$$

f) Le côté terminal contient le point P(-2, 1)



$$r^2 = (-2)^2 + 1^2$$

$$r^2 = 4 + 1$$

$$r = \sqrt{5}$$

$$\sin \theta = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos \theta = \frac{-2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$$

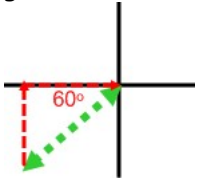
$$\tan \theta = \frac{-1}{2}$$

$$\operatorname{cosec} \theta = \frac{\sqrt{5}}{\sqrt{5}} = \sqrt{5}$$

$$\sec \theta = \frac{-\sqrt{5}}{2}$$

$$\operatorname{cotan} \theta = -2$$

g) $\theta = -120^\circ$



$$\sin \theta = \frac{-\sqrt{3}}{2}$$

$$\cos \theta = \frac{-1}{2}$$

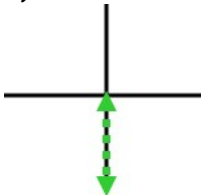
$$\tan \theta = \frac{2}{\sqrt{3}}$$

$$\operatorname{cosec} \theta = \frac{-2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$$

$$\sec \theta = -2$$

$$\operatorname{cotan} \theta = \frac{\sqrt{3}}{3}$$

h) $\theta = 270^\circ$



$$\sin \theta = -1$$

$$\cos \theta = 0$$

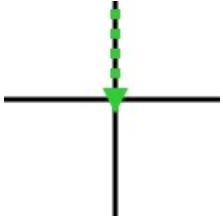
$$\tan \theta = \frac{-1}{0}; \text{ pas définie}$$

$$\operatorname{cosec} \theta = -1$$

$$\sec \theta = \frac{1}{0}; \text{ pas définie}$$

$$\operatorname{cotan} \theta = 0$$

i) $\theta \in [0^\circ, 180^\circ], \cos \theta = 0$



$\sin \theta = 1$

$\operatorname{cosec} \theta = 1$

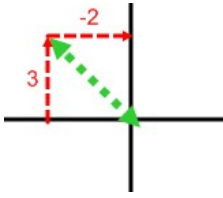
$\cos \theta = 0$

$\sec \theta = \frac{1}{0}$; pas définie

$\tan \theta = \frac{1}{0}$; pas définie

$\cotan \theta = 0$

j) $\sin \theta > 0, \cos \theta < 0, \tan \theta = \frac{-3}{2}$



$r^2 = 3^2 + 2^2$

$r^2 = 9 + 4$

$r = \sqrt{13}$

$\sin \theta = \frac{3}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$

$\operatorname{cosec} \theta = \frac{\sqrt{13}}{3}$

$\cos \theta = \frac{-2}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$

$\sec \theta = \frac{-\sqrt{13}}{2}$

$\tan \theta = \frac{-3}{2}$

$\cotan \theta = \frac{-2}{3}$

4. Détermine la valeur exacte de chaque expression.

a) $\cos 60^\circ + \sin 30^\circ$

b) $(\sec 45^\circ)^2$

c) $\operatorname{cosec} 60^\circ + \sec 210^\circ$

$\frac{1}{2} + \frac{1}{2} = 1$

$\left(\frac{1}{\cos 45^\circ}\right)^2 = \left(\frac{1}{\frac{\sqrt{2}}{2}}\right)^2$
 $= \left(\frac{2}{\sqrt{2}}\right)^2 = \frac{4}{2} = 2$

$\frac{1}{\sin 60^\circ} + \frac{1}{\cos 210^\circ}$
 $= \frac{1}{\frac{\sqrt{3}}{2}} + \frac{1}{-\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} - \frac{2}{\sqrt{3}} = 0$

5. Pour les angles suivants, détermine

i) un angle co-terminal positif, ii) un angle co-terminal négatif, iii) l'angle co-terminal principal iv) l'expression représentant tous les angles co-terminaux.



	a) -75°	b) 889°	c) $12\ 000^\circ$
i) un angle co-terminal positif,	$-75^\circ + 360^\circ = 285^\circ$	$889^\circ - 360^\circ = 529^\circ$	$12000^\circ - 360^\circ = 11640^\circ$
ii) un angle co-terminal négatif,	$-75^\circ - 360^\circ = -435^\circ$	$889^\circ - 360^\circ = 529^\circ$ $529^\circ - 360^\circ = 169^\circ$ $169^\circ - 360^\circ = -191^\circ$	$12000^\circ - 34 \times 360^\circ = -240^\circ$
iii) l'angle co-terminal principal	$-75^\circ + 360^\circ = 285^\circ$	$889^\circ - 360^\circ = 529^\circ$ $529^\circ - 360^\circ = 169^\circ$	$12000^\circ - 33 \times 360^\circ = 120^\circ$
iv) l'expression représentant tous les angles co-terminaux.	$285^\circ + 360^\circ n; n \in \mathbb{Z}$	$169^\circ + 360^\circ n; n \in \mathbb{Z}$	$120^\circ + 360^\circ n; n \in \mathbb{Z}$

6. (Parcours C). À l'aide du schéma ci-dessous, démontre que $\tan 15^\circ = \frac{1}{\sqrt{3} + 2}$.

